# Factorising the difference of two squares

The technique of factorising a quadratic expression has been explained on the leaflet *Factorising quadratic expressions*. There is a special case of quadratic expression known as **the difference of two squares**. This leaflet explains what this means and how such expressions are factorised.

## What is meant by the difference of two squares ?

A typical example of a quadratic expression which is the difference of two squares is $x^2 - 9$. Note that there is no $x$ term and that the number 9 is itself a square number. A square number is one which has resulted from squaring another number. In this case 9 is the result of squaring 3, $(3^2 = 9)$, and so 9 is a square number.

Hence $x^2 - 9$ is the difference of two squares, $x^2 - 3^2$.

When we try to factorise $x^2 - 9$ we are looking for two numbers which add to zero (because there is no term in $x$), and which multiply to give $-9$. Two such numbers are $-3$ and $3$ because

$$-3 \ + \ 3 \ = \ 0, \qquad \text{and} \qquad -3 \ \times \ 3 \ = \ -9$$

So

$$x^2 - 9 = (x - 3)(x + 3)$$

It is always the case that $x^2 - a^2$ factorises to $(x - a)(x + a)$.

> The difference of two squares, $x^2 - a^2$, always factorises to
>
> $$x^2 - a^2 = (x - a)(x + a)$$

**Example**

Factorise $x^2 - 25$.

Note that $x^2 - 25$ is the difference of two squares because 25 is a square number $(25 = 5^2)$. So we need to factorise $x^2 - 5^2$.

$$x^2 - 5^2 = (x - 5)(x + 5)$$

**Example**

Factorise $y^2 - 81$.

Note that $y^2 - 81$ is the difference of two squares because 81 is a square number $(81 = 9^2)$. So we need to factorise $y^2 - 9^2$.

$$y^2 - 9^2 = (y - 9)(y + 9)$$

**Exercises**

1. Factorise the following.

   a) $x^2 - 16$ b) $x^2 - 36$ c) $x^2 - 1$ d) $x^2 - 121$ e) $x^2 - 49$

## A different form

A slightly different form occurs if we now include a square number in front of the $x^2$ term:

**Example**

Suppose we wish to factorise $9x^2 - 16$.

Note that 9 is a square number, and so the term $9x^2$ can be written $(3x)^2$. So we still have a difference of two squares

$$(3x)^2 - 4^2$$

To factorise this we write

$$9x^2 - 16 = (3x - 4)(3x + 4)$$

Note that when multiplying-out the brackets the $x$ terms cancel out.

**Exercises**

2. Factorise the following.

   a) $9x^2 - 1$ b) $16x^2 - 9$ c) $49x^2 - 1$ d) $25x^2 - 16$

**Answers**

1. a) $(x - 4)(x + 4)$ b) $(x - 6)(x + 6)$ c) $(x - 1)(x + 1)$ d) $(x - 11)(x + 11)$ e) $(x - 7)(x + 7)$

2. a) $(3x + 1)(3x - 1)$ bj) $(4x + 3)(4x - 3)$ c) $(7x + 1)(7x - 1)$ d) $(5x + 4)(5x - 4)$